Advanced Macro - Problem set 5
Due date: classes on January, 6th

Problem 1 (1p) *Recursive solution to a standard optimal growth model usually does not have an analytical solution. An exception here, is a model with log preferences* $u(c) = \log(c)$ *and CD production* $f(k) = k^{\alpha}$, *with depreciation* $\delta = 1$. *Solve this problem recursively.*

- *a) Start with* $V_0(\cdot) = 0$ *and iterate on an operator defined on a Bellmana equation. After 2-3 iterations guess the functional form of a value function* V_t.
- *b) Using your answer from a) find the value function* V *(for infinite horizon problem)*
- *c) Find an optimal consumption policy (function). What is the optimal savings rate?*

Problem 2 (2p) *Consider an optimal growth model with* $E\sum_{t=0}^{\infty}\beta^t u(c_t, 1-l_t)$ *and production* $y_t = z_t f(k_t, l_t)$, *where* $z_t \in Z = \{\underline{z}, \overline{z}\}$ *and* $\overline{z} > \underline{z} > 0$, *with* $k_0 \geq 0$, z_0 *given. Let* $\Pi(\cdot|z_t)$ *denote a probability distribution (on* Z*) of a draw of* z_{t+1} *parameterized by* z_t. *Assume that capital belongs to* $[0, \overline{K}]$, *where* $\overline{K} = \overline{z}f(\overline{K}, 1)$. *Assume that* u *and* f *satisfy condition for recursive equilibrium existence.*

- *a) Write a Bellmana equation for a central planner's problem.*
- *b) Define a recursive competitive equilibrium for this economy with bonds.*

Problem 3 (2p) *Consider a household with preferences given by* $\sum_{t=0}^{\infty}\beta^t u(c_t)$, *where* $u : \mathbb{R}_+ \to \mathbb{R}$ *is bounded, continuous and strictly concave. Household begins as unemployed with stock* $a_0 > 0$ *of an asset. Each period unemployed household draws wage from* $[0, \overline{w}]$ *with a probability distribution* $F(w)$. *After accepting an offer in period* t *household starts working getting wage* w *from* $t+1$ *on. Employed household can loose a job with probability* λ *and starts next period as unemployed. When employed, wage* w *remains constant. There is no on the job search. Unemployed household gets a benefit* b. *There is a credit market for asset* a *with constant return* r, *i.e.* $a_{t+1} = (a_t - c_t + y_t)(1 + r)$, *where* y_t *is income (wage or benefit). There is no insurance market and* $a_t \geq 0$.

- *Identify state variables and write a Bellman equations for a recursive formulation for this maximization problem.*
- *What kind of properties need to by satisfied by a value function for a reservation wage strategy to be optimal? Show that value function indeed has these properties.*

Problem 4 (2p) *Consider an OLG model without elastic labor choice. Preferences are given by* $U(c_{1,t}) + \beta U(c_{2,t+1})$, *where* $U(c) = \ln c$ *and* $f(k) = k^{\alpha}$. *Assume that population grows at a constant rate* n, *i.e.* $N_{1,t} = (1 + n)N_{1,t-1}$ *and* $N_{1,t-1} = N_{2,t}$, *where* $N_{1,t}$ *is a number of young agents in period* t, *while* $N_{2,t}$ *no. of old people in* t, *and* N_t *is its total:* $N_t = N_{1,t} + N_{2,t} = N_{1,t} + \frac{1}{n+1}N_{1,t}$.

- *a) calculate the capital accumulation path for* $k_t = \frac{K_t}{N_t}$ *in ADCE*
- *b) prove that this economy has a unique steady state (pc), and convergence from* $k_0 > 0$ *is monotone. Derive an expression for a steady state (pc) capital level.*

Problem 5 (2p) *Consider an OLG economy as above. You are now asked to compare the ADCE under two taxing scenarios with balanced budget.*

- *i) Assume that government imposes lump sum tax* T *on each young generation and pays back* $T(1+n)$ *to every old.*
- *ii) Assume that now government imposes a lump sum tax* T *on each young generation, buys capital and gives* $(1 + r_{t+1})T$ *to every old.*

Answer (for both systems):

- *a) how such taxes affect (pc) capital accumulation path in an ADCE?*
- *b) how about steady state* k^* *pc?*
- *c) how such a tax system influences* K_t *on a BGP?*