1 Lecture 3: The New Keynesian Model

- The MIU model was not able to reflect features of real economies: lagged and prolonged reaction to shocks

- The MIU model showed full neutrality and superneutrality of money

- Moreover central banks have generally moved from controling monetary aggregates to controlling short-term interest rates.

- We need better models that can reproduce the basic features of the economy and inttroduce interest rates as monetary policy instrument.

- The New Keynesian Model goes in this direction

- It is the standard workhorse model of today's macroeconomists analyzing monetary policy and business cycles.

- Origin:

  - The NKM is a general equilibrium model (as MIU) and is based on the principle of microbased optimization. This can be traced back to the Lucas Critique and Real Business Cycle (RBC) economics of the 1980's and to the MIU model.

  - The standard MIU/ RBC model has been modified in two ways:

  1) the old postulate of the Keynesian school that nominal rigidities matter was incorporated

  2) the idea that central banks adjust interest rates (and not money) in reaction to deviations of inflation and output from targets (Taylor 1993) was incorporated.

- The NKM is the best (or at least most popular) we have, but we should be aware of its weaknesses. Economist constantly work on development of new models.

1.1 The model

Three basic modifications with respect to the MIU model:

- ignore endogenous capital adjustment as suggested by McCallum and Nelson (1999): it does not matter much for the analysis of business cycle fluctuations

- incorporate differentiated goods produced by monopolistically competitive firms (Dixit and Stiglitz 1977) facing constraints to price adjustments (Calvo 1983)

- represent monetary policy as setting the nominal interest rate in reaction to deviations of inflation and output from targets

1.1.1 Households maximize lifetime utility:

$$\max U = \sum_{i=0}^{\inf} \beta^i [\frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b}(\frac{M_{t+i}}{P_{t+i}})^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta}] \tag{1}$$

where M stands for nominal money balances, N for the work effort and C is a composite consumption good consisteing of differentiated products produced by monopolistically competitive firms.

$$C_t = [\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj]^{\frac{\theta}{\theta-1}} \tag{2}$$

where θ can be thought of as elasticity of substitution between the goods $c_{j.}$ The higher is θ the better substitutes are these goods.

To see this consider a two good case:

$$C_t = [c_1^{\frac{\theta-1}{\theta}} + c_2^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}} \tag{3}$$

Elasticity of substitution is the percentage change in good c_1 necessary to to keep utility (or production) constant when the other good c_2 is decreased by one percent.

$$e = \frac{\delta \ln(c_2/c_1)}{\delta \ln(TRS)} \tag{4}$$

$$TRS = \frac{\frac{\delta C}{\delta c_1}}{\frac{\delta C}{\delta c_2}} = (\frac{c_2}{c_1})^{\frac{1}{\theta}} \Rightarrow \ln(\frac{c_2}{c_1}) = \theta \ln(MRS) \tag{5}$$

hence

$$e = \theta \tag{6}$$

The household's problem can be solved in two steps. First, for any quantity of C_t the household chooses the chaepest composition of individual goods that give C_t.Next it chooses C_t, M_t and N_t.

First problem requires:

$$\min_{c_{j,t}} \int_0^1 p_{j,t} c_{j,t} dj \tag{7}$$

subject to:

$$[\int_0^1 c_{j,t}^{\frac{\theta-1}{\theta}} dj]^{\frac{\theta}{\theta-1}} = C_t \tag{8}$$

FOC:

$$p_{j,t} - \psi_t [(\int_0^1 c_{j,t}^{\frac{\theta-1}{\theta}} dj)^{\frac{1}{\theta-1}} c_{j,t}^{\frac{-1}{\theta}}] = 0 \tag{9}$$

so that:

$$c_{j,t} = (\frac{p_{j,t}}{\psi_t})^{-\theta} C_t \tag{10}$$

Substituting this into 2 we have:

$$C_t = (\psi)^{\theta} [\int_0^1 p_{jt}^{1-\theta} dj]^{\frac{\theta}{\theta-1}} \Rightarrow \psi = [\int_0^1 p_{jt}^{1-\theta} dj]^{\frac{1}{1-\theta}} \equiv P_t \tag{11}$$

which we define as our price index. Substituting into 10 we arrive at the demand function for good c_{jt}

$$c_{jt} = (\frac{p_{jt}}{P_t})^{-\theta} C_t \tag{12}$$

which deepends on the relative price level and aggregate spending. θ is the price elasticity of demand; the higher θ the more substitutable are goods. For $\theta \rightarrow$ **inf** a small change in the relative price brings about an infinite drop in demand.

To solve the consumer's problem we need a budget constraint:

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \frac{W_t}{P_t} N_t + \frac{M_{t-1}}{P_t} + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + D_t \tag{13}$$

where D_t stands for real dividends (profits) paid by firms to households.The lagrangian is:

$$\begin{aligned} L &= \sum_{i=0}^{\text{inf}} \beta^i [\frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} (\frac{M_{t+i}}{P_{t+i}})^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta}] + \\ & +\lambda [C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} - \frac{W_t}{P_t} N_t - \frac{M_{t-1}}{P_t} - (1 + i_{t-1}) \frac{B_{t-1}}{P_t} \end{aligned}$$

First order conditions for the consumer's problem are:

C_t :

$$C_t^{-\sigma} = \lambda_t \text{ and } \beta C_{t+1}^{-\sigma} = \lambda_{t+1} \tag{15}$$

$\frac{M_t}{P_t}$:

$$\gamma(\frac{M_t}{P_t})^{-b} - \lambda_t + \lambda_{t+1}\frac{P_t}{P_{t+1}} = 0 \tag{16}$$

$\frac{B_t}{P_t}$:

$$-\lambda_t + \lambda_{t+1}(1 + i_t)\frac{P_t}{P_{t+1}} = 0 \tag{17}$$

N_t:

$$-\chi N_t^{\eta} + \lambda_t \frac{W_t}{P_t} = 0 \tag{18}$$

These can be used to derive the basic equilibrium conditions. Substitute 15 into 17 to get the intertemporal Euler equation:

$$C_t^{-\sigma} = \beta(1 + i_t)\frac{P_t}{P_{t+1}}C_{t+1}^{-\sigma} \tag{19}$$

The oportunity cost of holding money is given (as in the MIU model) by (substitute into 16 : $\lambda_{t+1}$from 17 and for λ_t from 15):

$$\frac{\gamma(\frac{M_t}{P_t})^{-b}}{C_t^{-\sigma}} = \frac{i_t}{1 + i_t} \tag{20}$$

The labour - leisure choicee is given by (substitute 15 into 18):

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t} \tag{21}$$

1.1.2 Firms maximize profits

In the NKM we also have firms which maximize profits subject to technology:

$$y_{jt} = c_{jt} = Z_t N_{jt} \tag{22}$$

the demand curve (12) and price stickiness. Price stickiness (Calvo 1983) is imposed by assuming that only a fraction (1-ω) are allowed to reset their prices each period. The firm's problem can be written as:

$$\min_{N_t} \left(\frac{W_t}{P_t}\right) N_t + \phi_t (c_{jt} - Z_t N_{jt}) \tag{23}$$

where ϕ is the Lagrange multiplier and equals marginal cost, which can be seen from comparing the FOC:

$$\phi_t = \frac{W_t/P_t}{Z_t} \quad (24)$$

to:

$$\begin{aligned} MC &= \frac{\delta q}{\delta y} \text{ and } \; y = ZN \text{ and } q = \frac{W}{P}N = Wy/ZP \\ \text{so } MC &= \frac{W}{ZP} \end{aligned} \quad (25)$$

The firm chooses p_{jt} to maximize the expected value of the real present discounted value of future profits:

$$\max E_t \sum_{i=0}^{\text{inf}} [\omega^i \frac{1}{P_t}(p_{jt}c_{j,t+i} - P_{t+i}\phi_{t+i}c_{j,t+i}) \prod_{j=0}^{i} (\frac{1}{1+i_{t+j}})] \quad (26)$$

from 19 we have:

$$\prod_{j=0}^{i}(\frac{1}{1+i_{t+j}}) = \prod_{j=0}^{i}(\frac{C_{t+j+1}}{C_{t+j}})^{-\sigma}\beta\frac{P_{t+j}}{P_{t+j+1}} = = (\frac{C_{t+i}}{C_t})^{-\sigma}\beta^i\frac{P_t}{P_{t+i}} \quad (27)$$

The firms problem becomes:

$$\max E_t \sum_{i=0}^{\inf}\omega^i(\frac{C_{t+i}}{C_t})^{-\sigma}\beta^i[(\frac{p_{jt}}{P_{t+i}})c_{j,t+i} - \phi_{t+i}c_{j,t+i}] \quad (28)$$

Use 12 to eliminate c_{jt}. Also note that the firms are identical as to technology. So those firms that adjust their price at period t set the same price p_t^*.

$$\max E_t \sum_{i=0}^{\inf}\omega^i(\frac{C_{t+i}}{C_t})^{-\sigma}\beta^i[(\frac{p_t^*}{P_{t+i}})^{1-\theta} - \phi_{t+i}(\frac{p_t^*}{P_{t+i}})^{-\theta}]C_{t+i} \quad (29)$$

The first order condition for the optimal price is:

$$\sum_{i=0}^{\inf} \omega^i (\frac{C_{t+i}}{C_t})^{-\sigma} \beta^i [(1-\theta)(\frac{p_t^*}{P_{t+i}})^{-\theta} \frac{1}{P_{t+i}} +$$

$$+\theta \phi_{t+i} (\frac{p_t^*}{P_{t+i}})^{-\theta-1} \frac{1}{P_{t+i}}] C_{t+i} = 0 \qquad (30)$$

Rearanging terms we get:

$$\begin{aligned} & (1-\theta)(p^*)^{-\theta} \sum_{i=0}^{\inf} \omega^i (\frac{C_{t+i}}{C_t})^{-\sigma} \beta^i (P_{t+i})^{\theta-1} C_{t+i} \\ = & -\theta (p^*)^{-\theta-1} \sum_{i=0}^{\inf} \omega^i (\frac{C_{t+i}}{C_t})^{-\sigma} \beta^i \phi_{t+i} (P_{t+i})^{\theta} C_{t+i} \end{aligned}$$

Which, after dividing both sides by P_t and rearanging again gives:

$$\frac{p^*}{P_t} = \frac{\theta}{\theta - 1} \frac{\sum_{i=0}^{\inf} \omega^i (C_{t+i})^{1-\sigma} \beta^i \phi_{t+i} (\frac{P_{t+i}}{P_t})^{\theta}}{\sum_{i=0}^{\inf} \omega^i (C_{t+i})^{1-\sigma} \beta^i (\frac{P_{t+i}}{P_t})^{\theta-1}} \quad (31)$$

Consider the case when all firms can adjust their price ($\omega = 0$). Then we have:

$$p_t^* = \frac{\theta}{\theta - 1} \phi_t P_t \quad (32)$$

Thus the firm sets its price as a markup $\mu \equiv \frac{\theta}{\theta-1}$ over nominal marginal cost. The markup depends on elasticity of substitution between goods. The more monopolistic power the firms have ($\theta \rightarrow 0$) the higher will be the markup. With perfect competition ($\theta \rightarrow \inf$) price equals marginal cost.

1.2 Linear approximation

With $\omega \neq 0$ the optimal price will depend on expected marginal costs, expected prices and expected consumption. Linear approximation around the steady state yields (details in the appendix to Walsh (2003)):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \varepsilon_t \tag{33}$$

where $x_t \equiv \hat{y}_t - \hat{y}_t^f$ stands for the output gap (deviation of actual output from potential) and κ is a function of the deep parameters of the model. This is the **New Keynesian Phillips curve**. It links inflation to output and to expected inflation (firms are forward looking). This is a major building stone of the New Keynesian model.

The second building stone comes from the consumer's problem. Note that in this model $C_t = Y_t$. Approximate 19 around the zero inflation steady state:

$$\hat{y}_t = -\frac{1}{\sigma}(\hat{\imath} - E_t \pi_{t+1}) + E_t \hat{y}_{t+1} \tag{34}$$

Expressing this in terms of the output gap yields the **forward looking IS curve**:

$$x_t = -\frac{1}{\sigma}(\hat{\imath} - E_t \pi_{t+1}) + E_t x_{t+1} + u_t \qquad (35)$$

where u_t depends on the productivity shock z_t.

Equations 33 nad 35 augmented by a rule for setting the nominal interest rate (monetary policy) constitute the standard New Keynesian model.

1.3 Determinancy of equilibrium (Taylor principle)

- Does the model generate a unique and stable equilibrium?

- Uniqueness is an important issue, because rational expectation models can have several (even infinite number of) solutions.

- Stability is important as well: does the model return to equilibrium after a shock or does it explode? In other words, how can the central bank provide stability?

- The conditions for uniqeuness and stability of the RE solution were given by Blanchard and Kahn (1980).

Cosider our model 35 , 33 augmented by a simple monetary policy rule $\hat{\imath}_t = \rho_r \hat{\imath}_{t-1} + v_t$ so that monetary policy does not respond to what happens in the economy.

Write the model as:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \sigma^{-1} \\ 0 & 0 & \beta \end{bmatrix} \begin{bmatrix} \hat{\imath}_t \\ E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \rho_r & 0 & 0 \\ \sigma^{-1} & 1 & 0 \\ 0 & -\kappa & 1 \end{bmatrix} \begin{bmatrix} \hat{\imath}_{t-1} \\ x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} v_t \\ -u_t \\ -\varepsilon_t \end{bmatrix} \tag{36}$$

and multiply both sides by the inverse of the matrix on the left:

$$\begin{bmatrix} \hat{\imath}_t \\ E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = A^{-1} B \begin{bmatrix} \hat{\imath}_{t-1} \\ x_t \\ \pi_t \end{bmatrix} + A^{-1} \begin{bmatrix} v_t \\ -u_t \\ -\varepsilon_t \end{bmatrix} \qquad (37)$$

To do this recall that:

$$A^{-1} = \frac{1}{\det(A)} adj(A)$$

where the adjacent

$$
\begin{aligned}
adj(A) &= \begin{bmatrix} \begin{vmatrix} 1 & \sigma^{-1} \\ 0 & \beta \end{vmatrix} & -\begin{vmatrix} 0 & \sigma^{-1} \\ 0 & \beta \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \\ -\begin{vmatrix} 0 & 0 \\ 0 & \beta \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 0 & \beta \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} \\ \begin{vmatrix} 0 & 0 \\ 1 & \sigma^{-1} \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & \sigma^{-1} \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \end{bmatrix}' = \\
&= \begin{bmatrix} \beta & 0 & 0 \\ 0 & \beta & 0 \\ 0 & -\sigma^{-1} & 1 \end{bmatrix}' \qquad (38)
\end{aligned}
$$

Hence:

$$
M \equiv A^{-1}B = \begin{bmatrix} \rho_r & 0 & 0 \\ \sigma^{-1} & 1 + \frac{\kappa}{\sigma\beta} & -\frac{1}{\sigma\beta} \\ 0 & -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix} \qquad (39)
$$

- The Blanchard and Kahn condition requires the number of forward looking variables equal the number of eigenvalues of M outside the unit circle. We have 2 forward looking variables (with expectations): π and x. So we need 2 eigenvalues outside the unit circle.

- Let's check!

- What are eigenvalues? Eigenvalues of the matrix A are roots of the characteristic equation of the matrix $(A - rI)$. The characteristic equation is $\det(A - rI) = 0$.

- Eigenvalues are crucial i.a. for determining the stability of systems of difference equations.

- Example: take $\begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$ and calculate its eigenvalues.

$$\det\left(\begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix} - r\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

$$= \begin{vmatrix} 2-r & 2 \\ 2 & -1-r \end{vmatrix} = (2-r)(-1-r) - 4 = 0$$

$$r_1 = 3 \text{ and } r_2 = -2 \tag{40}$$

- The determinant of A_{3x3} is:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \tag{41}$$

- In our example the characteristic equation reduces to:

$$(\rho_r - r)\begin{vmatrix} 1 + \frac{\kappa}{\sigma\beta} - r & -\frac{1}{\sigma\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} - r \end{vmatrix} = 0 \tag{42}$$

- Assuming for simplicity $\beta = 1$ this becomes

$$(\rho_r - r)[(1 + \frac{\kappa}{\sigma} - r)(1 - r) - \frac{\kappa}{\sigma}] \tag{43}$$

- we can find the eigenvalues:

  $r_1 = \rho_r < 1$, $r_2 = \frac{2\sigma+\kappa+\sqrt{\Delta}}{2\sigma} > 1$ and $r_3 = \frac{2\sigma+\kappa-\sqrt{\Delta}}{2\sigma} < 1$ where $\Delta = 4\sigma\kappa + \kappa^2$.

- Only one eigenvalue (r_2) lies outside the unit circle. Thus the model has not a unique stable solution.

- Intuition: think of a shock raising inflation. If monetary policy does not react, real interest rates fall, deemand increases, and inflation rises further and further. The system explodes.

- Now think of a different monetary policy rule, where the central bank responds to observed inflation:

$$\hat{\imath}_t = \delta\pi_t + v_t \tag{44}$$

- This can be eliminated by substituting into 35 , so the system becomes in matrix notation:

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} = N \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1} v_t - u_t \\ 0 \end{bmatrix} \tag{45}$$

where:

$$N = \begin{bmatrix} 1 + \frac{\kappa}{\sigma\beta} & \frac{\beta\delta - 1}{\alpha\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix} \tag{46}$$

- Two eigenvalues are outside the unit circle if $\delta > 1$. This is called the Taylor principle: **monetary policy must react to rising inflation so that real interest rates rise and bring the system back to equilibrium. This is one of the basic laws of contemporaneous monetary economics.**

1.4 Simulations

- As with the MIU model we can run simulations to see how the model economy behaves after various shocks.

- We analyze three different shocks: supply shock, deemand shock and monetary policy shock.

- We use a hybrid NK model - the one derived above augmented by backward looking elements.

- This adds inertia to the model, but does not change the basic features.

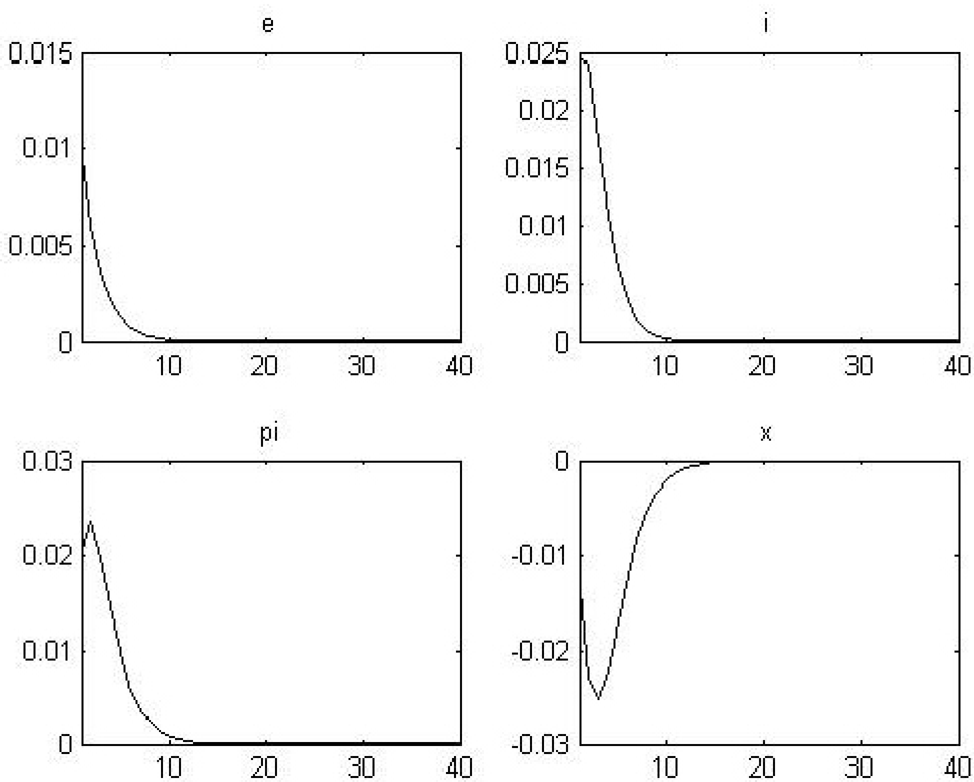

Figure 1: Reaction to supply shock

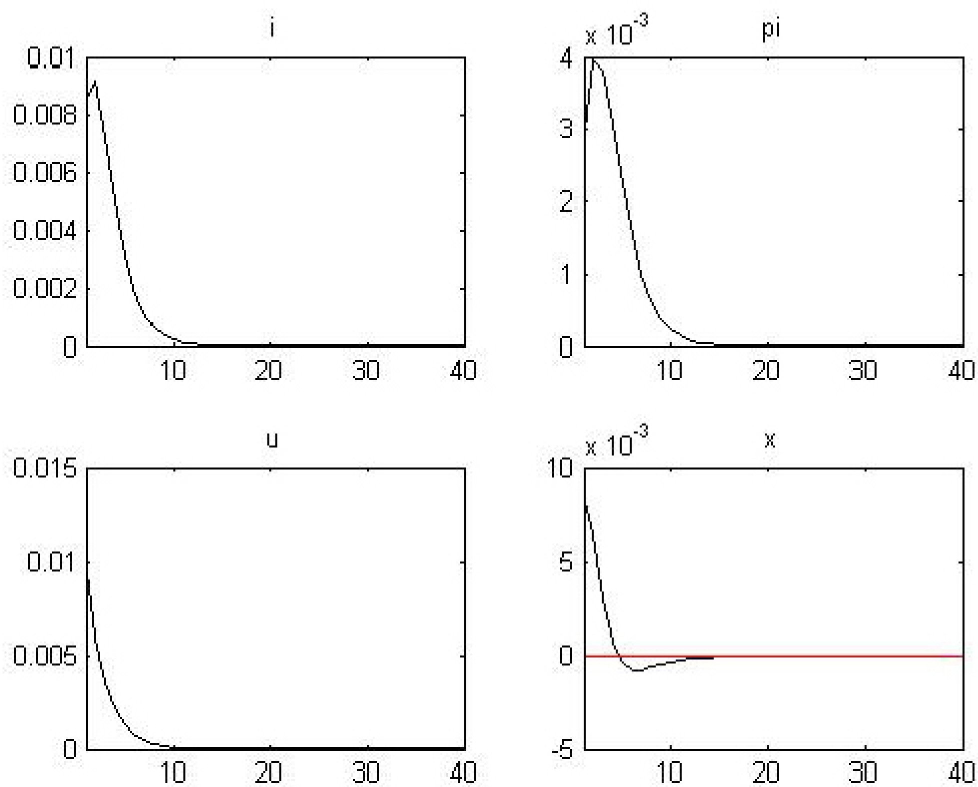

Figure 2: Reaction to demand shock

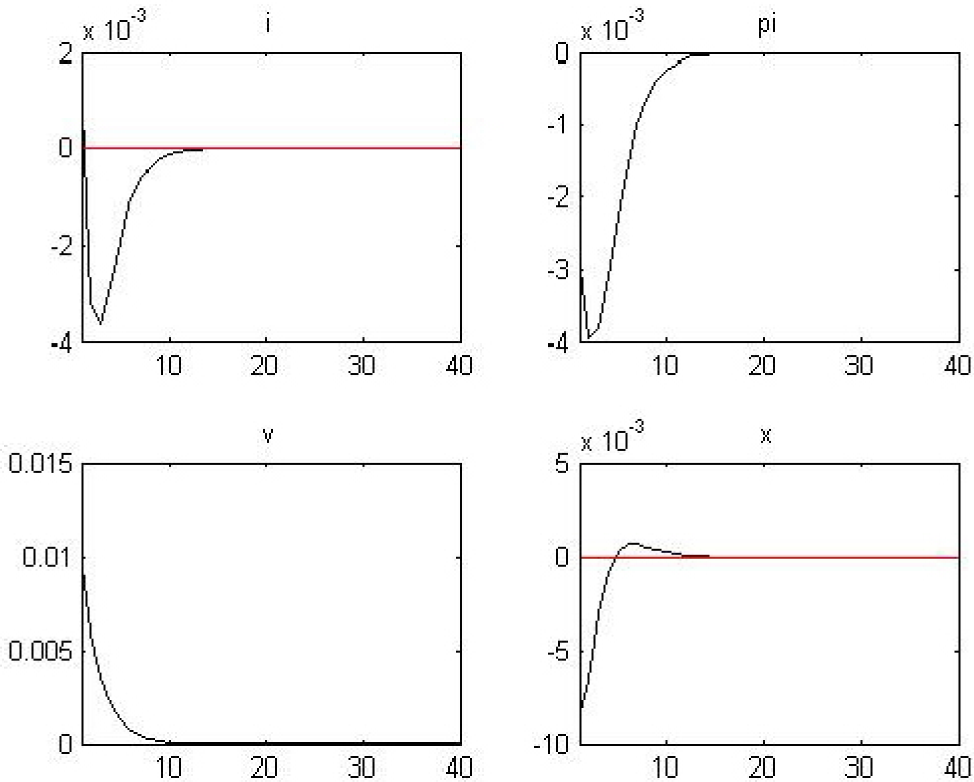

Figure 3: Reaction to monetary policy shock

1.5 Rules (commitment) versus discretion

- The rules vs. discretion debate was started with the paper of Kydland and Prescott (1997).

- They showed on the basis of a simple model that when there is a short term trade-off between output (employment) and inflation, monetary policymakers will try to boost output to maximize the social wealth function (or minimize the central bank's loss function).

- Forward-looking (rational) private agents will expect this and adjust expectations.

- As a result output will not be increased but inflation will be higher.

- We will show this result in the New Keynesian framework.

- Assume the central bank acts under discretion. This means that the bank does not make any commitment as to future policy and decides only about current interest rates.

- Hence, the central bank cannot affect expectations and its problem becomes static:

$$\min L = \frac{1}{2}[\pi_t^2 + \lambda(x_t - x^*)^2] \tag{47}$$

subject to 33 and 35 .

The Lagrangian is:

$$\frac{1}{2}[\pi_t^2 + \lambda(x_t - x^*)^2] + \mu[\pi_t - \beta E_t\pi_{t+1} - \kappa x_t - \varepsilon_t] +$$
$$+\theta[x_t + \frac{1}{\sigma}(\hat{\imath} - E_t\pi_{t+1}) - E_t x_{t+1} - u_t] \tag{48}$$

The FOCs are:

i :

$$\frac{\theta}{\sigma} = 0 \tag{49}$$

π :

$$\pi_t + \mu = 0 \quad (50)$$

x :

$$\lambda(x_t - x^*) - \mu\kappa + \theta = 0 \quad (51)$$

Hence $x_t = \frac{1}{\lambda}(\lambda x^* - \kappa\pi_t)$. Substitute this into 33 to get

$$\pi_t(1 + \frac{\kappa^2}{\lambda}) - \beta E_t \pi_{t+1} - \varepsilon_t - \kappa x^* = 0 \quad (52)$$

guess the solution to this difference equation takes the form

$$\pi_t = A(-\varepsilon_t - \kappa x^*) \text{ so that } E_t \pi_{t+1} = A(-\kappa x^*) \quad (53)$$

Substituting these into 52 yields:

$$A = \frac{\varepsilon_t + \kappa x^*}{(-\varepsilon_t - \kappa x^*)(\frac{\lambda+\kappa^2}{\lambda}) + \beta\kappa x^*} \tag{54}$$

Consider a benevolent central bank that, at time $t-1$ sets $x^* > 0$. Then expected value of period t inflation is:

$$\begin{aligned} E_{t-1}\pi_t &= E_{t-1}[\frac{-(\varepsilon_t + \kappa x^*)^2}{-(\varepsilon_t + \kappa x^*)(\frac{\lambda+\kappa^2}{\lambda}) + \beta\kappa x^*}] \\ &= \frac{\kappa x^*}{\frac{\lambda+\kappa^2}{\lambda} - \beta} > 0 \end{aligned} \tag{55}$$

- Several solutions to this problem have been suggested:

  - appoint a "conservative central banker" for whom $x^* = 0$ (Rogoff). Then 55 becomes:

$$E_t\pi_t = 0 \tag{56}$$

  - punish the central banker for too high inflation (New Zealand).

- let the central bank commit to a rule (i.e. say not only how you act today, but also how you will act in the future).

- Both, assuming $x^* = 0$ and commitment to a rule reduce the inflation bias to 0. But asuming $x^* = 0$ and acting under discretion results in higher volatility of the economy in response to shocks as compared to commitment (for details see Walsh pp. 523-529).

- Thus commitment is consideered (by academics) as the prefered solution.

- However central banks are reluctant to commit to follow a monetary policy rule.

1.6 The optimal rate of inflation

In the NKM inflation has two effects:

- Friedman (utility) effect - affects utility via reduced money demand (if money is in the utility function). We know from the MIU model that optimal inflation $\pi = -r$.

- Dispersion effect - Increases dispersion of relative prices bringing production and consumption allocation further away from optimum. Optimal inflation $\pi = 0$.

  The optimal rate of inflation is negative $-r < \pi^{OPT} < 0$. Details can be calculated.

  For optimal inflation to become positive (as most CBs target it) on has to add e.g. a nonnegativity constraint for nominal interest rates (e.g. Adam and Bini 2004).

1.7 Conclusions