New Keynesian model

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Flexible vs. sticky prices

Central assumption in the (neo)classical economics:
- Prices (of goods and factor services) are fully flexible
- An increase in money supply immediately increases prices 1:1
- Classical dichotomy: money is neutral and monetary policy has no real effects
- Consequences for models: we can abstract from money and nominal variables

(New) Keynesian economics:
- Prices are sticky, i.e. they adjust sluggishly to macroeconomic shocks (including monetary shocks)
- Classical dichotomy does not hold: monetary policy has real effects
- Also, additional propagation channels for other shocks
- Consequences for models: money and nominal variables important
Sticky prices: empirical evidence

- Price duration:
  - US: average time between price changes is 2-4 quarters (Blinder et al., 1998; Bils and Klenow, 2004; Klenow and Kryvstov, 2005)
  - Euro area: average time between price changes is 4-5 quarters (Rumler and Vilmunen, 2005; Altissimo et al., 2006)

- The higher inflation, the more frequently price changes occur

- Cross-industry heterogeneity
  - Prices of tradables less sticky than those of nontradables
  - Retail prices usually more sticky than producer prices
Why are prices sticky?

- Lucas (1972): imperfect information
  - Extensions: rational inattention (Sims, 2003; Mackowiak and Wiederholt, 2009), sticky information (Mankiw and Reis, 2007)
- Costs of changing prices (explicit or implicit):
  - Menu costs
  - Explicit contracts which are costly to renegotiate
  - Long-term relationships with customers
- 'Good' causes of price stickiness: in a stable economic environment agents trust in price stability
New Keynesian model - basic features

- General equilibrium model
- Two stages of production, at one of them firms are monopolistically competitive - can set their prices
- Firms are not allowed to reoptimize their prices each period - prices are sticky
- Hence, monetary policy has real effects and so needs to be described within the model
- In a nutshell - the RBC model with:
  - Sticky prices
  - Monetary authority operating via an interest rate feedback rule
  - Simplifications:
    - No capital accumulation - labour is the only factor
    - No trend productivity growth, only stationary stochastic shocks (hence, no need to normalize trending real variables)
Households I

- Rent labour (the only production factor) to firms
- Own firms and so get their profits $Div_t$
- Hold nominal bonds $B_t$, paying a nominal and risk-free (i.e. determined in the previous period) interest rate $R_{t-1}$ (expressed in gross terms)
- Make optimal consumption-savings (by adjusting bond holdings) and work-leisure decisions

Intertemporal budget constraint:

$$W_t L_t + Div_t + R_{t-1} B_t = P_t C_t + B_{t+1}$$  \hspace{1cm} (1)

where $P_t$ is the price level of consumption and $W_t$ is the nominal wage rate
Households maximize their expected lifetime utility:

\[ U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \]  \hspace{1cm} (2)

with instantaneous utility function \( u(C_t, L_t) \) given by:

\[ u(C_t, L_t) = \frac{C_t^{1-\theta}}{1-\theta} - \kappa \frac{L_t^{1+\varphi}}{1+\varphi} \]

The optimization is subject to the constraints:

- Budget constraint (1)
- Transversality condition:

\[ E_0 \lim_{t \to \infty} \left( \frac{B_t}{P_t} \prod_{s=1}^{t} \frac{1}{R_s} \right) \geq 0 \]  \hspace{1cm} (3)
Households’ optimization

- **Lagrange function:**

\[
LL = E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\theta}}{1-\theta} - \kappa \frac{L_t^{1+\varphi}}{1+\varphi} + \tilde{\lambda}_t \left[ W_t L_t + D_{ivt} - P_t C_t + R_{t-1} B_t - B_{t+1} \right] \right)
\]

- **First order conditions:**

\[
\frac{\partial LL}{\partial C_t} = 0 \implies C_t^{-\theta} = \tilde{\lambda}_t P_t \quad (4)
\]

\[
\frac{\partial LL}{\partial L_t} = 0 \implies \kappa L_t^\varphi = \tilde{\lambda}_t W_t \quad (5)
\]

\[
\frac{\partial LL}{\partial B_{t+1}} = 0 \implies \tilde{\lambda}_t = \beta E_t \left\{ \tilde{\lambda}_{t+1} R_t \right\} \quad (6)
\]
Firms

- Two stages of production:
  - Final-goods firms produce output by combining intermediate goods
  - Intermediate-goods firms produce using labour as the only input
- Contrary to the RBC model, final-goods production non-trivial since intermediate goods are not perfect substitutes. Therefore, the final output is not a simple sum of intermediate goods production.
Final-goods firms produce according to the CES production function (Dixit-Stiglitz aggregator):

\[ Y_t = \left( \int_0^1 Y_t(i) \frac{\phi-1}{\phi} \, di \right)^{\frac{\phi}{\phi-1}} \]  

(7)

where:
- The continuum of intermediate-goods firms (indexed by \( i \)) is normalized to 1.
- \( Y_t(i) \) is output produced by intermediate-goods firm \( i \).
- \( \phi > 1 \) is elasticity of substitution between individual intermediate goods.

Note: When \( \phi \to \infty \), \( Y_t \) is a simple sum of intermediate products (like in the RBC model, where all producers are perfectly competitive).
Final-goods firms II

- Maximization problem of final-goods firms:
  \[
  \max_{Y_t, Y_t(i)} \left\{ P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \right\}
  \]

  subject to constraint (7)

- Final-goods firms are competitive, so they maximize their profits by choosing the inputs \( Y_t(i) \), taking all prices \( (P_t(i) \) and \( P_t \) as given

- First order conditions:
  \[
  Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\phi} Y_t
  \]

  Equation (8) defines the demand for intermediate input \( i \)
Intermediate-goods firms I

- Linear production function in labour only:

\[ Y_t(i) = X_t L_t(i) \]  \hspace{1cm} (9)

- Productivity \( X_t \) is common to all firms and follows the first-order autoregressive process:

\[ \ln X_t = \rho \ln X_{t-1} + \varepsilon_t \]  \hspace{1cm} (10)

where: \( 0 \leq \rho < 1 \) and \( \varepsilon \sim iid(0, \sigma^2) \)

- Labour inputs rented from households, technology available for free
- Prices are set according to the Calvo (1983) mechanism
Maximization problem of intermediate-goods firm $i$:

$$\max_{P_t(i), Y_t(i), L_t(i)} \{ P_t(i) Y_t(i) - W_t L_t(i) \}$$

subject to the demand function (8) and the production function (9)

Maximization problem rewritten using the demand and production function constraints:

$$\max_{P_t(i)} \left\{ \left( P_t(i) - \frac{W_t}{X_t} \right) \left( \frac{P_t(i)}{P_t} \right)^{-\phi} Y_t \right\}$$

Each intermediate-goods firms takes the economy-wide wage rate $W_t$ and output $Y_t$ as given
First-order condition:

\[ P_t(i) = \frac{\phi}{\phi - 1} \frac{W_t}{X_t} \]  

(11)

Note that \( \frac{W_t}{X_t} \) is marginal cost.

So, imperfectly competitive intermediate-goods firms set their prices as a (constant) mark-up over marginal costs, where the mark-up equals to \( \frac{\phi}{\phi - 1} \).

Note that since neither mark-ups nor marginal costs are firm-specific, all intermediate-goods firms choose the same prices.
Price setting with sticky prices I

- **Calvo scheme:**
  - Each period a constant proportion $1 - \gamma$ ($0 < \gamma < 1$) of randomly selected intermediate-goods firms is allowed to reset their prices.
  - The remaining intermediate-goods firms have to keep their prices unchanged.
- Firms allowed to reset their price take into account that they may not be allowed to do so in the future.
- The probability that in period $t + s$ the price of intermediate-goods firm $i$ is still $P_t(i)$ equals $\gamma^s$.
- The expected time of a price remaining fixed equals $(1 - \gamma)^{-1}$.
Maximization problem of intermediate-goods firm $i$:

$$
\max_{P_t(i)} \left\{ E_t \sum_{s=0}^{\infty} \tilde{\lambda}_{t+s} \beta^s \gamma^s \left( P_t(i) - \frac{W_{t+s}}{X_{t+s}} \right) \left( \frac{P_t(i)}{P_{t+s}} \right)^{-\phi} Y_{t+s} \right\}
$$

Note:

- Profit maximization is dynamic: firms must take into account that they may not have a chance to reset their prices in the future
- Firms are owned by households, so they discount the utility value of their future profits by the discount factor $\beta$

First-order condition:

$$
E_t \sum_{s=0}^{\infty} \tilde{\lambda}_{t+s} \beta^s \gamma^s \left( P_t(i) - \frac{\phi}{\phi - 1} \frac{W_{t+s}}{X_{t+s}} \right) \left( \frac{P_t(i)}{P_{t+s}} \right)^{-\phi} Y_{t+s} = 0 \quad (12)
$$
First-order condition (12) is the same for each firm allowed to reset its price.

Therefore, all firms allowed to reoptimize at time \( t \) choose the same price, which we denote by \( \tilde{P}_t \).

The aggregate price level \( P_t \) is then:

\[
P_t = \left( \int_0^1 P_t(i)^{1-\phi} \, di \right)^{1/(1-\phi)} = (1 - \gamma) \tilde{P}_t^{1-\phi} + \gamma P_{t-1}^{1-\phi} \frac{1}{1-\phi} \tag{13}
\]

where the first equality follows from (8).
Monetary policy

- Prices are sticky, so monetary policy has real effects
- Monetary authorities set the short-term (one period) nominal interest rate according to the Taylor-like feedback rule (see Taylor, 1993):

\[ R_t = R + a_\pi (\pi_t - \bar{\pi}) + a_y (\ln Y_t - \ln Y) \tag{14} \]

where:
- \( R = \frac{\bar{\pi}}{\beta} \) is steady state nominal (gross) interest rate
- \( \pi = \frac{P_t}{P_{t-1}} \) is (gross) inflation and \( \bar{\pi} \) is the target (steady state) inflation
- \( Y \) is steady-state output
- \( a_\pi > 1, a_y \geq 0 \)

- Central bank can completely stabilize inflation by responding very aggressively to deviations of inflation from the target (i.e. by choosing a very large value for \( a_\pi \))
General equilibrium

- Market clearing conditions:
  - Output produced by firms must be equal to households’ total spending (consumption):
    \[ Y_t = C_t \] (15)
  - Labour supplied by households must be equal to labour demanded by firms:
    \[ L_t = \int_0^1 L_t(i) di = \int_0^1 \frac{Y_t(i)}{X_t} di = \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\phi} Y_t X_t di = \Delta_t \frac{Y_t}{X_t} \] (16)

where: \( \Delta_t = \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\phi} di \geq 1 \) is a measure of price dispersion
\( (\Delta_t = 1 \iff \forall i : P_t(i) = P_t) \)

- Proceeding similarly to (13) one can show that:
  \[ \Delta_t = (1 - \gamma) \left( \frac{\tilde{P}_t}{P_t} \right)^{-\phi} + \gamma \left( \frac{P_{t-1}}{P_t} \right)^{-\phi} \Delta_{t-1} \] (17)
Equilibrium dynamics can be summarized by 9 equations (4), (5), (6), (12), (13), (14), (15), (16) and (17), as well as the transversality condition (3).

They describe the evolution of 9 endogenous variables: $C_t$, $L_t$, $W_t$, $\tilde{\lambda}_t$, $Y_t$, $L_t$, $P_t$, $\tilde{P}_t$ and $R_t$.

This system can actually be reduced to just 5 endogenous variables: $Y_t$, $P_t$, $\tilde{P}_t$, $\Delta_t$ and $R_t$.

The only exogenous driving force in the model is stochastic productivity $X_t$ defined by (10).

In principle, the New Keynesian model usually includes also other shocks.
Due to nonlinearities and presence of expectations, the model does not have a closed-form solution.

Standard technique: log-linear approximation of the model equations around the (non-stochastic) steady-state.

Non-stochastic steady-state: no productivity shocks and all variables in the model constant.
New Keynesian Phillips curve

Log-linearized (12) and (13) imply the following New Keynesian Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \gamma)(1 - \beta \gamma)}{\gamma} mc_t$$

(18)

where:

- $mc_t$ is log deviation of real marginal cost (i.e. $\frac{W_t}{X_t P_t}$) from its steady-state value
- we assumed for simplicity that the target (and so steady-state) inflation rate is zero
Log-linearized (4), (6) and (15) imply the following New Keynesian IS curve:

\[ y_t = E_t y_{t+1} - \frac{1}{\theta} (R_t - R - E_t \pi_{t+1}) \]  

(19)

where:

- \( y_t = \ln Y_t - \ln Y \) is log deviation of output from its steady-state value
- we assumed for simplicity that the target (and so steady-state) inflation rate is zero
Standard calibration:

- $\theta = 2$ (more reasonable than 1 if there is no capital)
- $\beta = 0.99$
- $\kappa = 1$
- $\varphi = 1$
- $\phi = 6$ (implies a steady-state mark-up of 20%)
- $\gamma = 0.75$
- $a_\pi = 1.5$, $a_y = 0.5$ (Taylor, 1993)
- $\rho = 0.95$, $\sigma = 0.01$

$\kappa$ and $\phi$ do not appear in the log-linearized version of the model
Technology shock

Notes: grey line - flexible prices ($\gamma \to 0$); black line - sticky prices (our baseline model); time unit - quarters; all variables expressed as percentage (inflation and interest rate - percentage point) deviations from their steady-state values
Monetary shock I

- Monetary shock: an additive stochastic component $RR_t$ in the interest rate rule, so that (14) becomes:

$$R_t = R + a_\pi (\pi_t - \bar{\pi}) + a_y (\ln Y_t - \ln Y) + RR_t$$

- $RR_t$ follows a first-order autoregressive process:

$$RR_t = \rho_R RR_{t-1} + \varepsilon_{R,t}$$

where $0 \leq \rho_R < 1$
Monetary shock II

Notes: $\rho_R = 0.90$; time unit - quarters; all variables expressed as percentage (inflation and interest rate - percentage point) deviations from their steady-state values.
Government spending shock I

- Government spending $G_t$ fully financed by lump sum taxes $V_t$ levied on households, so that $G_t = V_t$ holds every period.
- Modifications to the model:
  - Households’ budget constraint (1) becomes:
    \[ W_tL_t + \text{Div}_t + R_{t-1}B_t = P_tC_t + B_{t+1} + V_t \]
  - Goods market clearing condition (15) becomes:
    \[ Y_t = C_t + G_t \]
- It is easy to verify that first-order conditions of households’ maximization problem (4)-(6) remain unchanged.
- We assume that government spending is stochastic and follows a first-order autoregressive process:
  \[ \ln G_t = (1 - \rho_G)G + \rho_G \ln G_{t-1} + \varepsilon_{G,t} \]
  where:
  - $0 \leq \rho_G < 1$
  - $G$ is the steady state level of government spending.
Notes: $\rho_G = 0.95$; government spending share in output is 20%; time unit - quarters; all variables expressed as percentage (inflation and interest rate - percentage point) deviations from their steady-state values
Shock decomposition in a standard medium-sized closed-economy model (Smets and Wouters, 2007)
Shock decomposition in a standard medium-sized closed-economy model (Smets and Wouters, 2007)
Shock decomposition in a standard medium-sized open-economy model (Christoffel et al., 2008)
Shock decomposition in a model with financial frictions (Christiano et al., 2010)
The role of expectations

- Equation (18) implies that current inflation is affected by inflation expectations
- Modern monetary policy: management of expectations
- Woodford: *For not only do expectations about policy matter, (...) but very little else matters*
Optimal monetary policy

- Equation (16) implies that price dispersion (i.e. $\Delta_t > 1$) is costly.
- Price dispersion can be eliminated if the central bank chooses to stabilize inflation at zero (i.e. sets the inflation target to zero and responds very aggressively to any deviations from the target).
- Hence, a policy strictly stabilizing inflation can replicate the flexible price equilibrium.
- However, monetary policy may face a trade-off between stabilizing inflation and keeping output at a desired (not constant, in general) level.
- This trade-off vanishes if:
  - steady state output is efficient (i.e. distortions related to monopolistic competition are eliminated, e.g. by proper subsidies to firms)
  - there are no cost-push shocks (i.e. shocks to the Phillips curve)
- In this case perfect price stability is optimal.
Very simple dynamic stochastic general equilibrium model (DSGE) with monopolistic competition and sticky prices

Monopolistic power of firms $\implies$ decentralized allocations are not Pareto optimal (production not at an efficient level)

Price stickiness restores the role of monetary policy:
- Monetary policy has real effects (affects output, consumption, real wages)
- The case for price stabilization: price stability eliminates price distortion
- Pursuing strict price stabilization is optimal if steady state distortions (due to monopolistic competition) are eliminated (e.g. by production subsidies)

The workhorse model in central banks