1. Consider a standard real business cycle model. The equilibrium of such a model can be characterized by the following set of log-linearized equations:

\[ k_{t+1} = \beta^{-1}k_t + \frac{b}{\alpha} [A_t + (1 - \alpha)l_t - c_t] + \delta c_t \]

\[ -\theta c_t = -\theta E_t \{c_{t+1}\} + bE_t \{A_{t+1} + (\alpha - 1)k_{t+1} + (1 - \alpha)l_{t+1}\} \]

\[ \varphi l_t + \theta c_t = A_t + \alpha k_t - \alpha l_t \]

\[ A_{t+1} = \rho A_t + \varepsilon_{t+1} \]

where \( \varepsilon_t \sim N(0, \sigma^2) \) and \( b = \beta^{-1} - 1 + \delta \).

(a) Write the model in a matrix form as in formula (1) from the auxiliary material "Solving DSGE models...". Give matrices \( A_1, A_0 \) and \( \gamma \) as a solution. Can this system be solved using the standard (i.e. based on the Jordan decomposition) Blanchard-Kahn algorithm? Why? (0.5p)

(b) Use the third of the above four equations to eliminate \( l_t \). Write the thus obtained system of three equations in a matrix form, giving matrices \( A_1, A_0 \) and \( \gamma \). (0.5p)

(c) Assume \( \beta = 0.99, \alpha = 0.33, \theta = 2, \delta = 0.025, \varphi = 1, \rho = 0.95 \) and \( \sigma = 0.01 \). Solve numerically the model obtained in (b), writing it as \( W_t = \Phi W_{t-1} + \Psi Z_t \), where \( W_t = \begin{bmatrix} A_t & k_t & c_t \end{bmatrix}' \). Give \( \Phi \) and \( \Psi \) as a solution. HINT: You can use a modified version of the Matlab code given in the auxiliary material "Implementing the Blanchard-Kahn algorithm...". (1p)

(d) Use the production function \( y_t = A_t + \alpha k_t + (1 - \alpha)l_t \) to augment \( W_t \) with \( y_t \) and calculate the new \( \Phi \) and \( \Psi \). Assuming that the economy at time \( t = 0 \) is in the steady state, calculate the response of output \( y_t \) to a typical positive technology shock (i.e. \( \varepsilon_1 = \sigma, \varepsilon_t = 0 \) for \( t > 1 \)) for the first 100 periods. Print or sketch a chart as a solution, reporting the values at \( t = 1, 4, 8, 16 \). (1p)
2. Consider the standard small-scale log-linearized New Keynesian model given by the following equations:

\[ x_t = E_t \{x_{t+1}\} - \frac{1}{\theta} (R_t - E_t \{\pi_{t+1}\} - r^*_n) \]

\[ \pi_t = \beta E_t \{\pi_{t+1}\} + \kappa x_t \]

where \( \kappa > 0, \theta > 0, 0 < \beta < 1 \), while \( r^*_n \) is the natural interest rate that is a function of the exogenous disturbances and model parameters only. Consider a monetary policy rule of the form:

\[ R_t = r^*_n + \phi_\pi \pi_t + \phi_x x_t \]

Find all pairs of non-negative \( \phi_\pi \) and \( \phi_x \) for which the model has a unique solution. What can you say about inflation and the output gap under this policy rule? (2p)

3. Consider the New Keynesian model in the following log-linearized form:

\[ x_t = E_t \{x_{t+1}\} - \frac{1}{\theta} (R_t - E_t \{\pi_{t+1}\}) + \varepsilon_t^d \]

\[ \pi_t = \beta E_t \{\pi_{t+1}\} + \kappa x_t + \varepsilon_t^s \]

where \( \kappa = \frac{(1-\gamma)(1-\beta_\gamma)(\theta+\phi)}{\gamma} \), The demand shock \( \varepsilon_t^d \) and the supply (cost-push) shock \( \varepsilon_t^s \) follow independent AR(1) processes with autoregressive coefficients \( \rho_d \) and \( \rho_s \), and standard deviations of innovations \( \sigma_d \) and \( \sigma_s \), respectively.

(a) Assume that the central bank acts such that inflation \( \pi_t \) is zero at all times. How does the output gap \( x_t \) and the nominal interest rate \( R_t \) respond to demand and supply shocks? As your answer derive the formulas in which both variables depend on \( \varepsilon_t^d, \varepsilon_t^s \) and the model parameters only. (1p)

(b) Do the same exercise for an alternative monetary policy variant that implies zero output gap \( x_t \) at every period. (1p)

(c) Can the formulas for \( R_t \) derived in (a) and (b) be considered as monetary policy rules that ensure perfect stabilization of, respectively, inflation and the output gap? If not, give examples of such rules that achieve these goals. (0.5p)

(d) Based on the results obtained in (a) and (b), derive the formulas for ergodic variances of the output gap \( E(x^2) \) and inflation \( E(\pi^2) \) for both monetary policy variants (such that these moments depend on the model parameters only). (1p)

(e) Assume that the government subsidies production such that social welfare is proportional to \( -E(\pi^2) - \kappa \phi E(x^2) \). Show for what values of \( \kappa \) strict stabilization of inflation is better than strict stabilization of the output gap. Which of these extreme policies will be preferred if the degree of price stickiness is low, and which if it is high? (0.5p)