Macroeconomics II
Winter semester 2011/2012
Warsaw School of Economics
Instructor: Marcin Kolasa
marcin.kolasa(at)sgh.waw.pl

Assignment 2

Due date: 19 December 2011, 1.30 p.m.

1. Consider the Solow model with human capital. Assume that the production function is of the Cobb-Douglas form: $Y_t = K_t^{\alpha_K} H_t^{\alpha_H} (A_t L_t)^{1-\alpha_K-\alpha_H}$, where $\alpha_K, \alpha_H > 0$ and $\alpha_K + \alpha_H < 1$.

   (a) Derive the long-run solution of the model by writing the formula for output on the balanced growth path (i.e. for Y_t when the economy is in the steady state) as a function of the model's exogenous variables (A_t, L_t) and parameters (s_K, s_H, n, g, d_K, d_H, α_K, α_H) only. (**1p**)

   (b) Assume the following parameter values: $n = 0.01$, $g = 0.03$, $d_K = 0.1$, $d_H = 0.05$, $\alpha_K = 0.3$, $\alpha_H = 0.4$, $s_K = 0.2$ and $s_H = 0.25$. The initial conditions are: $A_0 = L_0 = K_0 = H_0 = 1$. Use a spreadsheet to give the following numbers: (i) the growth rates of output and output per capita in $t = 1$, (ii) the period number t^* in which the gap between output per unit of effective labour y_t and its steady-state level falls below a half of the initial value, (iii) the growth rates of output and output per capita in t^*. (do not submit or print the whole spreadsheet, just provide the solution) (**2p**)

   (c) Investigate how t^* defined in 1.b depends on α_K and α_H by allowing them to vary between 0.2, 0.3 and 0.4, i.e. provide 9 estimates of t^* for 9 different combinations of α_K and α_H. (**1p**)

2. Consider a similar model as in 1, except that now no raw labour is used in production, i.e. $\alpha_K + \alpha_H = 1$ and so the production function can be written as $Y_t = AK_t^{\alpha} H_t^{1-\alpha}$, where A is a constant (i.e. there is no technological progress).

   (a) Assume $A = 0.5$, $n = 0.01$, $d_K = 0.08$, $d_H = 0.05$, $\alpha = 0.3$, $s_K = 0.2$, $s_H = 0.25$, $L_0 = H_0 = 1$ and $K_0 = 0.25$. Use a spreadsheet to generate the path of the output per capita growth rate for $t = 0, 1, ..., 100$. Plot and print it. (do not submit or print the whole spreadsheet, only the chart) (**1p**)

   (b) Assume now that the initial level of physical capital K_0 is such that the net rates of return on both types of capital are equalized. Write the formula that implicitly defines K_0 as a function of the model parameters and other initial conditions only. Evaluate it assuming the values as in 2.a. HINT: The net rate of return on capital is its marginal product less the depreciation rate. To solve for K_0, use numerical methods (e.g. the Goal Seek tool in Excel), setting the starting value to 0.5. (**1p**)

(c) Assume now that s_K is no longer constant but evolves in a way ensuring that the ratio of physical to human capital is constant and equal to the value calculated in 2.b for $t = 0$. What does this assumption imply for the net rate of return on each type of capital (justify your answer)? Use a spreadsheet to generate the path of the output per capita growth rate for $t = 0, 1, ..., 100$. What can you say about the growth rate of output per capita in this economy? (do not submit or print the whole spreadsheet, just print the chart or sketch it by hand) (**2p**)

(d) Consider an identical economy as in 2.a, except that its initial physical capital is two times lower, i.e. $K_0 = 0.1$. Use a spreadsheet to generate the path of output per capita in this country relative to that in 2.a. (do not submit or print the whole spreadsheet, only the chart) Will output per capita levels in these two economies ever converge? What would your answer be for a standard Solow model (or its human capital extension considered in 1) and why? (**1p**)

3. Consider the two-period business cycle model. Assume that the production function is of a Cobb-Douglas form $Y = AK^{\alpha}L^{1-\alpha}$ ($Y' = A'K'^{\alpha}L'^{1-\alpha}$), where $0 < \alpha < 1$, while households' utility function is given by $u = \ln C + a \ln l$ ($u' = \ln C' + a \ln l'$), where $a > 0$.

   (a) Write the system of 11 equations that jointly determine the equilibrium values of L, L', K', I, C, C', Y, Y', w, w' and r. (**1p**)

   (b) Assume that capital fully depreciates ($d = 1$) and there are no government purchases ($G = G' = 0$). Reduce the system of 11 equations to 3, jointly determining the equilibrium values of L, L' and K'. (**1p**)

   (c) Solve analytically the system from 3.b, i.e. write L, L' and K' as functions of the model parameters and exogenous variables (i.e. A, A' and K) only (**1p**)

   (d) Assume $\alpha = 0.4$, $a = 1$, $\beta = 0.95$, $A = A' = 1$, $K = 0.1$. Calculate the equilibrium values of all variables listed in 3.a. HINT: If you do not obtain $r = 11.3\%$, double-check your calculations. (**1p**)

   (e) Calculate the percentage effect on all model endogenous variables listed in 3.a of a temporary increase in productivity, i.e. A going up by 5%. (**1p**)

   (f) Calculate the percentage effect on all model endogenous variables listed in 3.a of a permanent increase in productivity, i.e. both A and A' going up by 5%. (**1p**)